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Effective Monopole Potential
for SU(2) Lattice Gluodynamics
in Spatial Maximal Abelian Gauge

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ABSTRACT

We investigate the dual superconductor hypothesis in finite-temperature $SU(2)$ lattice gluodynamics in the *Spatial* Maximal Abelian gauge. This gauge is more physical than the ordinary Maximal Abelian gauge due to absence of non-localities in temporal direction. We show numerically that in the Spatial Maximal Abelian gauge the probability distribution of the abelian monopole field is consistent with the dual superconductor mechanism of confinement: the abelian condensate vanishes in the deconfinement phase and is not zero in the confinement phase.

The dual superconductor hypothesis of color confinement [1] in gluodynamics has been confirmed by various lattice calculations [2] in the so-called Maximal Abelian (MaA) projection [3]. This hypothesis is based on a partial gauge fixing of a non-abelian group up to its abelian subgroup. After gauge is fixed abelian monopoles arise due to singularities in the gauge fixing conditions [4]. If monopoles are condensed then the vacuum of gluodynamics behaves as a dual superconductor and the electric charges (quarks) in such a vacuum are confined.

The MaA projection on the lattice is defined by the condition [3]:

$$\max_{\Omega} R_{\mathrm{MaA}}[U^{\Omega}], \qquad R_{\mathrm{MaA}}[U] = \sum_{l} \mathrm{Tr}[\sigma_3 U_l^+ \sigma_3 U_l], \quad l = \{x, \mu\}, \tag{1}$$

where the summation is over all lattice links and $U_{x,\mu}$ are the lattice $SU(2)$ gauge fields.

The gauge fixing condition (1) contains time components of the gauge fields, $U_{x,4}$, therefore abelian operators in the MaA projection correspond to nonlocal in time operators in terms of the original $SU(2)$ fields $U_{x,\mu}$. To show this let us consider the expectation value of the $U(1)$ invariant operator $\mathcal{O}$ in the Maximal Abelian gauge [5, 6]:

$$< \mathcal{O} >_{\mathrm{MaA}} = \frac{1}{\mathcal{Z}_{\mathrm{MaA}}} \int \mathcal{D}U \, \exp\{-S[U] + \lambda R_{\mathrm{MaA}}[U]\} \, \Delta_{FP}[U; \lambda] \, \mathcal{O}(U), \quad \lambda \to +\infty, \tag{2}$$

where $\mathcal{Z}_{\mathrm{MaA}} =< 1 >_{\mathrm{MaA}}$ is the partition function in the fixed gauge. $\Delta_{FP}[U; \lambda]$ is the Faddeev–Popov determinant:

$$1 = \Delta_{FP}[U; \lambda] \cdot \int \mathcal{D}\Omega \, \exp\{\lambda R_{\mathrm{MaA}}[U^{\Omega}]\}, \quad \lambda \to +\infty.$$

Shifting the fields $U \to U^{\Omega^+}$ in eq.(2) and integrating over Ω both in the nominator and denominator, we get:

$$< \mathcal{O} >_{\mathrm{MaA}} =< \tilde{\mathcal{O}}_{\mathrm{MaA}} >, \quad \tilde{\mathcal{O}}_{\mathrm{MaA}}(U) = \frac{\int \mathcal{D}\Omega \, \exp\{\lambda R_{\mathrm{MaA}}[U^{\Omega}]\} \, \mathcal{O}(U^{\Omega})}{\int \mathcal{D}\Omega \, \exp\{\lambda R_{\mathrm{MaA}}[U^{\Omega}]\}},$$

$\tilde{\mathcal{O}}_{\mathrm{MaA}}$ is the $SU(2)$ invariant operator. Since $\lambda \to +\infty$ we can use the saddle point approximation to calculate $\tilde{\mathcal{O}}_{\mathrm{MaA}}$:

$$\tilde{\mathcal{O}}_{\mathrm{MaA}}(U) = \frac{\sum_{j=1}^{N(U)} Det^{\frac{1}{2}} M_{\mathrm{MaA}}[U^{\Omega^{(j)}}] \, \mathcal{O}(U^{\Omega^{(j)}})}{\sum_{k=1}^{N(A)} Det^{\frac{1}{2}} M_{\mathrm{MaA}}[U^{\Omega^{(k)}}]}, \tag{3}$$

here $\Omega^{(j)}$ corresponds to the N–degenerate global maxima of the functional $R_{\mathrm{MaA}}[U^{\Omega}]$ with respect to the regular gauge transformations Ω: $R_{\mathrm{MaA}}[U^{\Omega^{(j)}}] = R_{\mathrm{MaA}}[U^{\Omega^{(k)}}]$, $j, k = 1, \dots, N$. The matrix M_{MaA} is the Faddeev–Popov operator [5]:

$$M_{\mathrm{MaA}}^{x,a;y,b}[U] = \frac{\partial^2 R_{\mathrm{MaA}}(U^{\Omega(\omega)})}{\partial \omega^a(x) \, \partial \omega^b(y)}\bigg|_{\omega=0},$$

$\Omega(\omega) = \exp\{i\omega^a T^a\}$, $T^a = \sigma^a/2$ are the generators of the gauge group, σ^a are the Pauli matrices.

Since the gauge fixing functional R_{MaA} contains time components of the gauge field U then the operator $\tilde{\mathcal{O}}(U)$ is non-local in time. For time-nonlocal operators there are obvious difficulties with the transition from the Euclidean to Minkowski space–time. Thus there are problems with physical interpretation of the results obtained for abelian operators in the MaA projection.

The MaA gauge condition can be easily modified to overcome this time-non-locality problem. The corresponding gauge condition is given by:

$$\max_\Omega R_{\mathrm{SMaA}}[\hat{U}^\Omega], \qquad R_{\mathrm{SMaA}}[U] = \sum_{\mathbf{l}} \mathrm{Tr}[\sigma_3 U_{\mathbf{l}}^+ \sigma_3 U_{\mathbf{l}}], \quad \mathbf{l} = \{x, i\}, \quad i = 1, 2, 3, \quad (4)$$

where the summation is taken only over the spatial links $\mathbf{l}$. We refer to this projection as the Spatial Maximal Abelian (SMaA) projection[1]. In the SMaA gauge the gauge invariant operator (3) is local in time.

In this paper we study the abelian monopole condensate Φ_c in SMaA projection. To calculate Φ_c we need the monopole creation operator $\Phi_{\mathrm{mon}}(x)$. This operator was found by Fröhlich and Marchetti [8] for compact electrodynamics and was studied numerically in Refs. [9]. The Fröhlich–Marchetti operator was generalized to the abelian projection of lattice $SU(2)$ gluodynamics in Refs. [10] where it was found that in the MaA projection the monopole field is condensed in the confinement phase and Φ_c vanishes in the deconfinement phase[2].

The construction [10] of the monopole creation operator for an arbitrary Abelian projection is the following. We parametrize the $SU(2)$ link matrix in the standard way: $U_{x\mu}^{11} = \cos\phi_{x\mu}\, e^{i\theta_{x\mu}}$; $U_{x\mu}^{12} = \sin\phi_{x\mu}\, e^{i\chi_{x\mu}}$; $U_{x\mu}^{22} = U_{x\mu}^{11*}$; $U_{x\mu}^{21} = -U_{x\mu}^{12*}$; $0 \le \phi \le \pi/2$, $-\pi < \theta, \chi \le \pi$. The plaquette action in terms of the angles ϕ, θ and χ can be written as follows: $S_P = \frac{1}{2}\mathrm{Tr}\, U_1 U_2 U_3^+ U_4^+ = S^a + S^n + S^i$, where $S^a = \cos\theta_P \cos\phi_1 \cos\phi_2 \cos\phi_3 \cos\phi_4$, S^n and S^i describe the interaction of the fields θ and χ and self–interaction of the field χ [5], here the subscripts $1, ..., 4$ correspond to the links of the plaquette: $1 \to \{x, x+\hat{\mu}\}, ..., 4 \to \{x, x+\hat{\nu}\}$. For a fixed abelian projection, each term S^a, S^n and S^i is invariant under the residual $U(1)$ gauge transformations: $\theta_{x\mu} \to \theta_{x\mu} + \alpha_x - \alpha_{x+\hat{\mu}}$, $\chi_{x\mu} \to \chi_{x\mu} + \alpha_x + \alpha_{x+\hat{\mu}}$.

The operator $\Phi_{\mathrm{mon}}(x)$ creates the monopole at the point x on the dual lattice with a cloud of dual photons, it is defined as follows [10]:

$$\Phi_{\mathrm{mon}}(x) = \exp\left\{\sum_P \tilde{\beta}\left[-\cos(\theta_P) + \cos(\theta_P + W_P(x))\right]\right\}, \quad (5)$$

where $\tilde{\beta} = \beta\, \cos\phi_1 \cos\phi_2 \cos\phi_3 \cos\phi_4$, W_P is defined as follows [8]: $W_P = 2\pi\delta\Delta^{-1}(D_x - \omega_x))$. The integer valued 1-form $^*\omega_x$ represents the Dirac string attached to the

monopole [8] and satisfies the equation: $\delta^* \omega_x = {}^* \delta_x$. The function $D_x = d_{(3)} \Delta_{(3)}^{-1} \delta_x$ represents the cloud of dual photons. Here ${}^* \delta_x$ is the lattice δ–function, it equals to unity at the site x of the dual lattice and is zero at the other sites; $d_{(x)}$ and $\Delta_{(3)}^{-1}$ are the lattice derivative and the inverse Laplace operator on three-dimensional time-slice which includes the point x.

We study the monopole creation operator Φ_{mon} in the SMaA projection (4) on the lattices $4 \cdot L^{33}$, for $L = 8, 10, 12, 14, 16$. We extrapolate the value of the monopole condensate to the infinite spatial volume ($L \to \infty$) since near the deconfinement phase transition there are strong finite volume effects. We also impose the so–called C– periodic boundary conditions in space for the gauge fields, since the periodic boundary conditions are forbidden due to the Gauss law: we input a magnetic charge into the finite box. The C–periodic boundary conditions for the nonabelian gauge fields [13] correspond to anti–periodic spatial boundary conditions for abelian fields. In the case of $SU(2)$ gauge group the C–periodic boundary conditions are almost trivial: on the boundary we have $U_{x,\mu} \to \Omega^+ U_{x,\mu} \Omega$, $\Omega = i\sigma_2$. Note, that the gauge–fixing functionals for MaA (1) and for SMaA (4) gauges are invariant under this transformation.

The effective potential $V(\Phi)$ for the monopole field is defined via the probability distribution of the operator Φ_{mon}, Ref. [9, 10]:

$$V(\Phi) = -ln(< \delta(\Phi - \Phi_{mon}(x)) >). \qquad (6)$$

We calculated numerically this potential by the Monte-Carlo method. We generate the gauge fields by the standard heat bath method taking 2000 update sweeps to thermalize the system at each value of β. The number of the gauge fixing iterations is defined by the standard condition [14]: the iterations are stopped when the matrix of the gauge transformation $\Omega(x)$ becomes close to the unit matrix: $\max_x \{1 - \frac{1}{2} Tr\, \Omega(x)\} \le 10^{-5}$. We check that more accurate fixing of the SMaA gauge (4) does not change our results. To calculate the probability distribution for each value of β at the lattice of definite size, we use 100 gauge field configurations separated by 300 Monte Carlo sweeps. Then for each field configuration we calculate the value of the monopole creation operator (5) at 20 randomly chosen lattice points.

In Fig. 1 we show the effective potential (6) for confinement phase, the lattice is $4 \cdot 12^3$. This potential corresponds to the Higgs type potential[4]. The value of the monopole field, Φ_c at the minimum of the minimum is equivalent to the value of the monopole condensate.

The minimum of the potential, Φ_c, vs. inverse lattice size, $1/L$, is shown in Fig. 2 for two values of β. We fit the data for $\Phi_c(L)$ by the formula $\Phi_c = AL^\alpha + \Phi_c^{inf}$, where A, α and Φ_c^{inf} are the fitting parameters. We find that the best fit gives $\alpha = -1$ within the statistical errors.

Fig. 3 shows the value of the monopole condensate, extrapolated to the infinite spatial volume, Φ_c^{inf}. We conclude from Fig. 3 that in the SMaA projection the infinite–

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[3]These lattices correspond to finite temperature field theory, $T = 1/4a$, a is the lattice spacing.

[4]In Fig. 1 the only right half of the potential is shown due to positivity of the monopole operator (5), see Refs. [10] for a discussion.

volume condensate Φ_c^{inf} vanishes at the point of the phase transition[5], $\beta = \beta_c$. This in the SMaA projection the confining vacuum of $SU(2)$ gluodynamics behaves as a dual superconductor.

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M.N. Chernodub and M.I. Polikarpov, in *"Confinement, Duality and Nonperturbative Aspects of QCD"*, p.387, Ed. by Pierre van Baal, Plenum Press, 1998; `hep-th/9710205`;
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[5]It is well known that for $\beta = \beta_c \approx 2.3$ ($\beta > \beta_c$) the $SU(2)$ gauge field is in the confinement (deconfinement) phase on the lattice $L^3 \times 4$

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E.–M. Ilgenfritz, H. Markum, M. Müller–Preussker, W. Sakuler and S. Thurner, *Progr. Theor. Phys. Suppl.* **131** (1998) 353; `hep-lat/9804031`.

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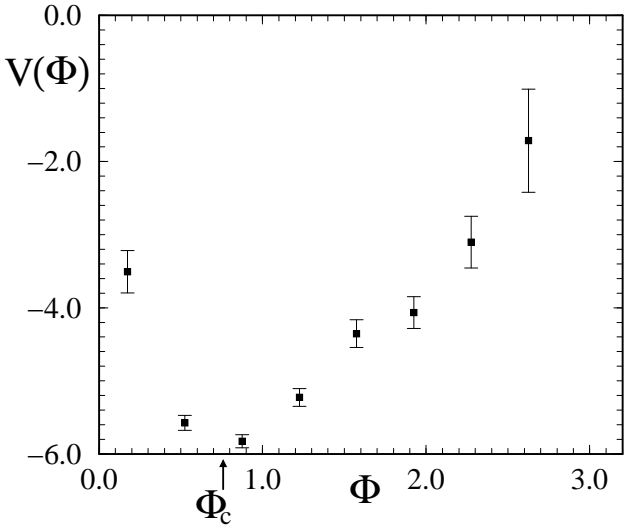G. I. Poulis, *Phys. Rev.* **D56**, 161 (1997).

Figures

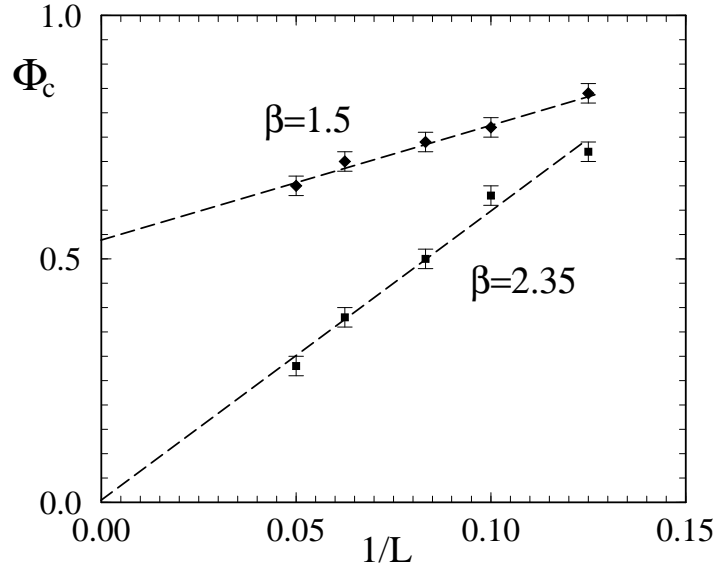Figure 1: Effective potential $V(\Phi)$, eq.(6), for confinement phase, $\beta = 1.5$.

Figure 2: Finite volume monopole condensate, Φ_c, *vs.* inverse spatial size of the lattice, $1/L$, at $\beta = 1.5$ and $\beta = 2.35$.
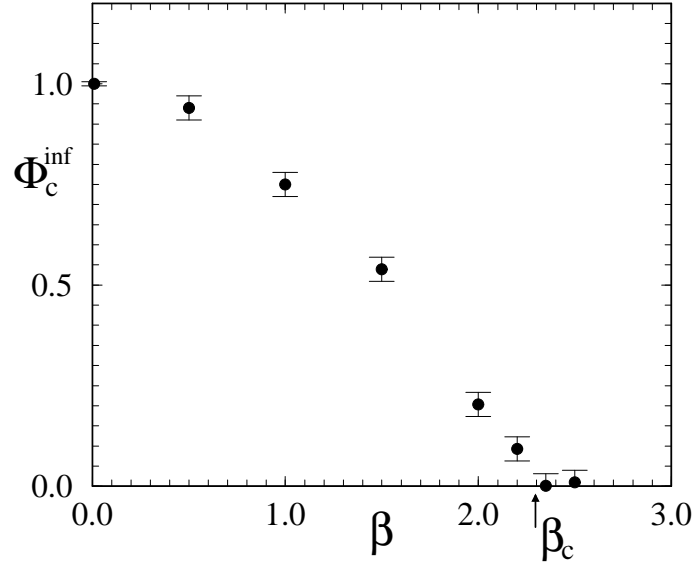

Figure 3: Monopole condensate extrapolated to the infinite volume, Φ_c^{inf}, *vs.* β. The phase transition is at $\beta = \beta_c \approx 2.3$.